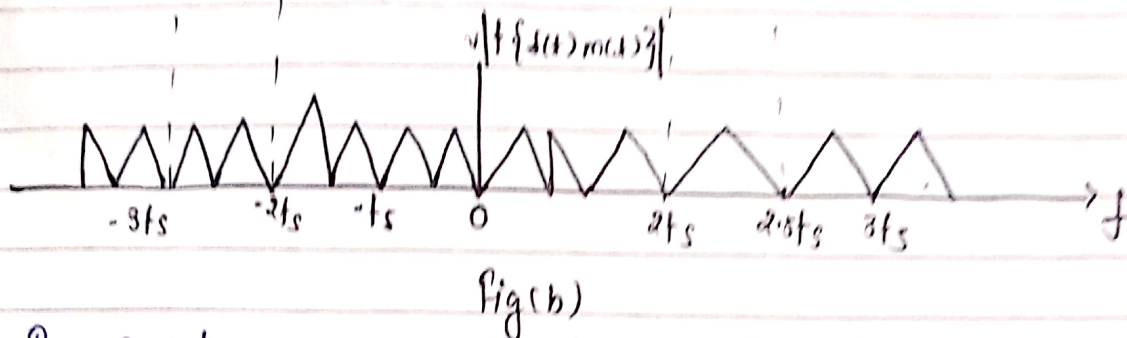
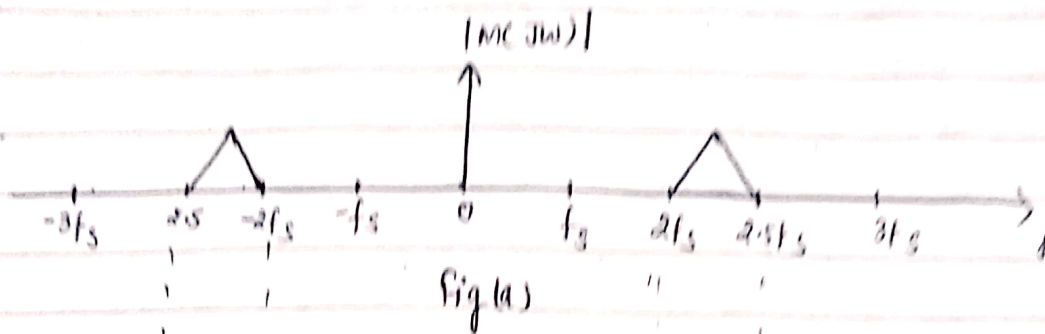


Band Pass Sampling - Theorem 4

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Band Pass Signals - 4



- a) The spectrum of a band pass signal
b) The spectrum of the sampled band pass signal.

The sampling freq. $f_s \geq 2f_m$ needs only when if the lowest freq. spectral component of original signal $m(f)$ is zero i.e. when $f_2 = 0$. But if $f_2 \neq 0$ then sampling freq. f_s need not to be larger than $2(f_m - f_2)$ i.e. $f_s = 2(f_m - f_2)$

To establish sampling theorem from band pass signal we initially assumed that the freq. f_2 may be the integral multiple of f_s i.e. $f_2 = n f_s$ where n is an integer. As shown in fig (a) with $n=2$ i.e. the lowest freq. f_2 consider with the n^{th} harmonic of the sampling freq. While fig (b) represents the spectral pattern of the sampled signal. Thus from fig, it is

clear that if the sampled signal $S(t) m(t)$ passed through a band pass filter with sharp cut off at $f_2 = 2f_s$ and $f_m = 2.5f_s$, then the original signal will be recovered exactly.

Now consider the general case i.e. neither f_m nor f_2 is a harmonic of sampling freq f_s .

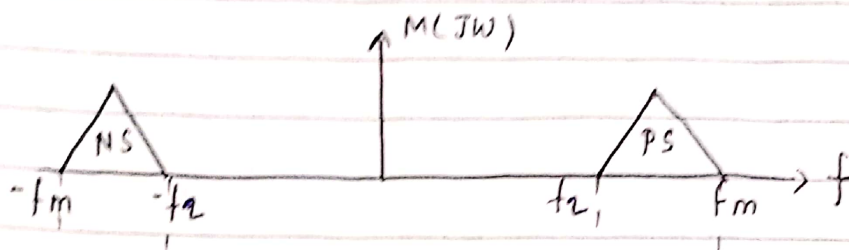


fig (c)

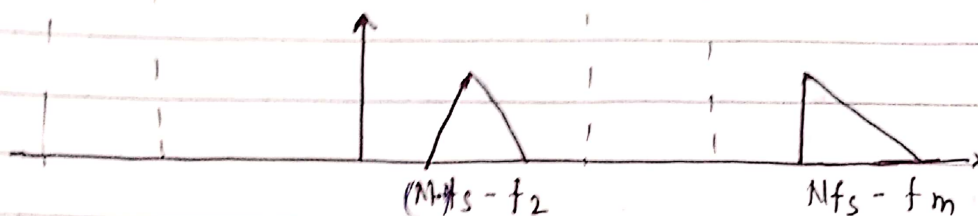


fig (d)

c) represent the original signal $m(t)$ which is band limited b/w f_2 and f_m

Now if we select the minimum value of the sampling freq. is to be $f_s = 2(f_m - f_2) = 2B$ (let) where $B = \text{Band width}$

Thus, shifted P_s pattern will not overlap with P_s but the shifted pattern of N_s will also generate a series of shifted pattern to the left and to the right. The left shifting of N_s can not cause an overlap of P_s but the right shifting of N_s might be cause an overlap

fig (d) represents, the right shifted pattern of N_s ,

with due to the $(N-1)^{\text{th}}$ and N^{th} harmonic of the sampling wave form. Thus to avoid overlap it is necessary that

$$(N-1)f_s - f_L \leq f_L \quad \text{--- (1)}$$

and

$$Nf_s - f_L \geq f_M \quad \text{--- (2)}$$

By taking $f_M - f_L = B$

eq (1) becomes

$$(N-1)f_s \leq 2(f_M - B) \quad \text{--- (a)}$$

$$\text{and } Nf_s \geq 2f_M \quad \text{--- (b)}$$

let

$$\frac{f_M}{B} = K \quad (\text{some constant})$$

then eq (a) & (b) becomes

$$f_s \leq 2B \left(\frac{K-1}{N-1} \right) \quad \text{--- (c)}$$

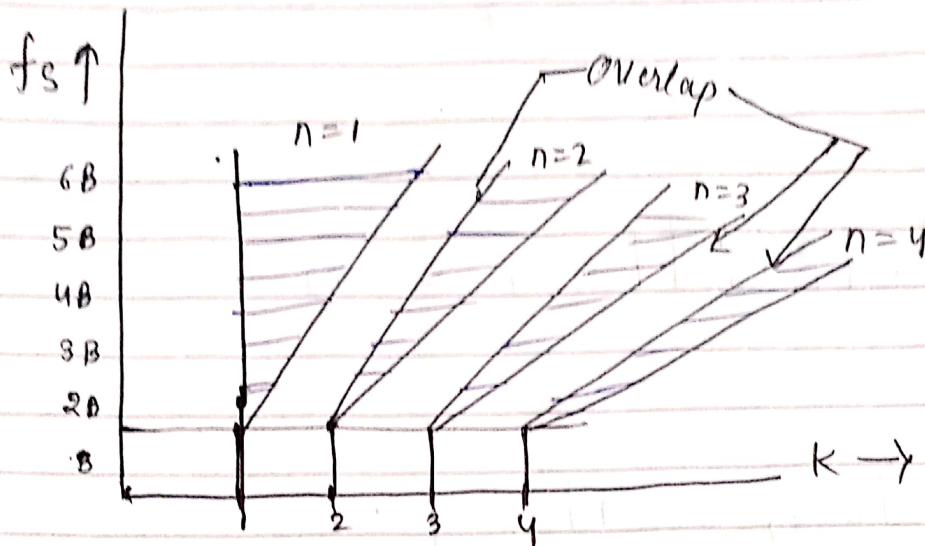
$$f_s \geq 2B \left(\frac{K}{N} \right) \quad \text{--- (d)}$$

eq (c) & (d) are the conditions which must be observed to avoid an overlap on PS. Due to the symmetry of the spectrum the same conditions are observed to avoid an overlap of PS on NS.

Now consider a particular case, in which the base band signal as $f_L = 2.5 \text{ kHz}$ and $f_M = 3.5 \text{ kHz}$.

Thus $B = f_M - f_L = 1 \text{ kHz}$ and $K = \frac{3.5}{1} = 3.5$

Now we plot a graph b/w f_s and k from eqn (c) and (d) [use several value of k and N]



In fig shaded region are the regions where conditions are satisfied while in the unshaded region the conditions are not satisfied and overlap will occur.

Natural Sampling - It is inconvenient to use impulses to sample the signal the reason is that the intensity of the original signal at the receiving end is infinitely small due to which such signals will may be lost in background noise. To avoid this we use natural sampling. In case of natural sampling the sampling wave form $s(t)$ consist of a train of pulses having duration T and separated by the sampling time T_s . The fig shows the